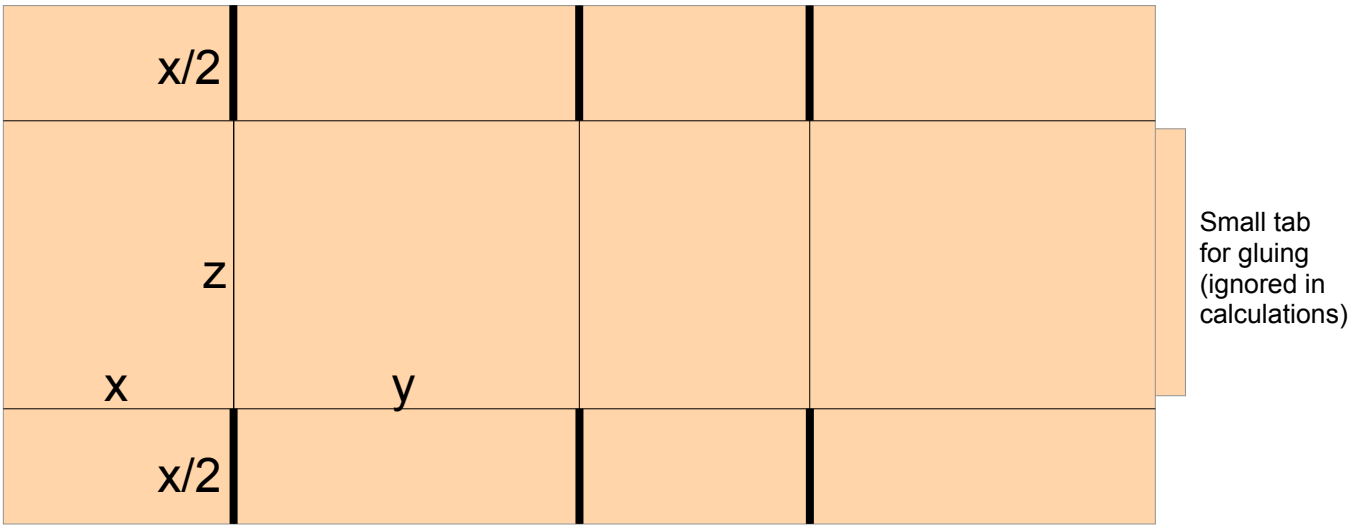
Name:_____

Calculus November Assignment: Cardboard Box Problem.

Take any old box out of the package room and measure the width, length and height. (We'll call them x, y and z.) Now carefully take the box apart so that it is a single rectangular sheet of cardboard with six cuts and a eleven folds in it. I've done this in the diagram below.



a unfolded cardboard box that has dimensions x, y & z when folded

Our task is to determine the dimensions, (x, y & z) that will give us the most volume out of the least amount of cardboard.

1) Using x, y & z, write a formula for the volume of the folded box.

2) Using x, y & z, write a formula for the area of the rectangular sheet of cardboard.

3) Rearrange the volume formula to get z on one side all by itself.

4) Substitute for z in the area formula. You should now have area as a function of x, y and V.

--Now we need to do some multivariable calculus, which sounds scary but it's not--

5) Find the partial derivative of the area with respect to x, $(\frac{\partial A}{\partial x})$. This just means: suppose that x is the only variable and all the other symbols are just numbers.

•Examples: The derivative of $f(x)=5\pi x^4$ is $f'(x)=20\pi x^3$. The partial derivative of $P=x^2y+xy^3$ with respect to x is $\frac{\partial P}{\partial x}=2xy+y^3$. The partial derivative of $P=x^2y+xy^3$ with respect to y is $\frac{\partial P}{\partial y}=x^2+3xy^2$.

6) Now find the partial derivative of the area with respect to y.

7) In order to minimize the area, we want to set each partial derivative equal to zero. We now have two equations and two unknowns. You should be able to take one of these and solve it for x or for y and get something reasonable on the other side.

8) Substitute that result into the other equation.

9) Things probably look pretty messy at this point, but there is more hope than you think. The quadratic formula will help us out. Hint: instead of $ax^2+bx+c=0$ you're looking to get something like $a(y^3)^2+b(y^3)+c=0$. Then the quadratic formula solves for y^3 instead of x .

10) If you can get a value for x or y in terms of V, then you can plug that into the formula from step 7 to get the other one, and then you can plug into step 3 and find z. And then you have the dimensions of a box that minimize the cardboard to enclose a given volume.

11) To finish off: Write the formula that gives x as a function of V. Write two formulas that give y as a function of x, and z as a function of x.

Bonus) On a sheet of graph paper, create a diagram like the one above, where x is six boxes and y and z are however long the formulas say they have to be. Cut the diagram out and fold it into a box.